Summary of
General Education Assessment Implementation
in the
Mathematics Department

October 17, 2011

This document summarizes

• the efforts to date of the Mathematics Department towards implementing assessment of general education learning outcomes in courses designated as Mathematical Modeling courses, and
• data collected during assessments given in the spring semester, 2011.

This document was created at the request of VPUE Sonya Stephens, in preparation for her report with GenEd co-chair Munirpallam Venkataramanan to the BFC on October 18, 2011.

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I. Definition of mathematical modeling (MM) courses and respective GenEd learning objectives

*From the General Education website and BFC circular B16-2011:*

1. Mathematical modeling courses
   a. are mathematics courses that either are required for students in the natural and mathematical sciences or address problems through mathematical models;
   b. emphasize mathematical rigor and abstraction, fundamental mathematical skills, and college-level mathematical concepts and techniques;
   c. teach how to develop mathematical models and draw inferences from them;
   d. include a full semester or equivalent of frequent and regular assignments that provide practice in mathematical modeling and mathematical techniques.

Problems providing modeling practice
i. are phrased with limited use of mathematical notation and symbols;
ii. require a formulation step on the part of the student;
iii. require college-level mathematical techniques leading from the formulation to the conclusion;
iv. have a conclusion that involves discovery or interpretation.

2. Courses approved for the Mathematical Modeling requirement must demonstrate and provide a system for consistency in instruction and in assessment of student achievement.

3. Courses approved for the mathematical modeling requirement should engage students with mathematical concepts and techniques that prepare them for a variety of possible future courses and degrees.

4. A course used to satisfy the Mathematical Modeling Foundations requirement may not double-count toward the Breadth of Inquiry Natural and Mathematical Sciences requirement.

The following courses will apply to the IU Bloomington GenEd Mathematical Modeling requirement if taken in Summer 2011, Fall 2011, or Spring 2012.
MATH-A 118  Finite Mathematics for the Social and Biological Sciences (3 cr.)
MATH-D 116 and D 117  Introduction to Finite Mathematics I and II (2 cr. + 2 cr.)*
MATH-J 113  Introduction to Calculus with Applications (3 cr.)
MATH-M 118  Finite Mathematics (3 cr.)
MATH-S 118  Honors Finite Mathematics (3 cr.)
MATH-M 119  Brief Survey of Calculus I (3 cr.)
MATH-M 211  Calculus I (4 cr.)
MATH-M 213  Accelerated Calculus (4 cr.)
MATH-V 118  Finite Mathematics with Applications: Finite and Consumer Math
MATH-V 118  Finite Mathematics with Applications: Finite Mathematics for the Social and Biological Sciences

* Note: MATH-D 116–D 117 is a two-course sequence. Credit is not given for D 116 until D 116 is completed with a minimum grade of C– and D 117 is completed with a passing grade.

Math M118, M119, J113, D116-17, and M211 are multi-section courses with a designated experienced faculty member serving as coordinator. Enrollments for e.g. M118, M119 typically run in the low thousands.
Learning Objectives

Students proficient in Mathematical Modeling should demonstrate the ability to

1. create mathematical models of empirical or theoretical phenomena in domains such as the physical, natural, or social science;

2. create variables and other abstractions to solve college-level mathematical problems in conjunction with previously-learned fundamental mathematical skills such as algebra;

3. draw inferences from models using college-level mathematical techniques including problem solving, quantitative reasoning, and exploration using multiple representations such as equations, tables, and graphs;

4. take an analytical approach to problems in their future endeavors.

A passing grade in an approved course is required to show proficiency in mathematical modeling under the General Education curriculum.
II. Outline of Mathematics Department efforts

At the request of chair Kevin Zumbrun, an internal Mathematics Department committee (called MathGenEd below) was created in the Fall of 2010 to implement assessment in mathematics courses for general education. For academic years 2010-11, this comprised: former chair David Hoff, current Director of Undergraduate Studies Chris Connell (chair), Senior Lecturer Tracy Whelan, and GenEd MM subcommittee chair and internal Mathematics/GenEd liason Kevin M. Pilgrim. In 2011-12, Connell (chair), Whelan, and Pilgrim continue to serve.

It was decided to pilot implementation, ahead of the required schedule, in M118 and M119 for Spring 2011.

The MathGenEd committee recognized that GenEd MM LO #4, “...take an analytic approach to future endeavors”, while desirable, was nonetheless not measurable. It also felt that attempting to assess proficiency in GenEd MM LOs #s 1, 2, and 3 separately would be artificial. It decided that each assessment instrument created would measure proficiency simultaneously in all three GenEd MM LOs, and that different instruments would measure different course-specific LOs.

Soliciting input from faculty, it created course-specific learning objectives. These were phrased to reflect the nature of the content of the courses, to be measurable, and to be maximally consistent with the GenEd MM course learning outcomes.

Meeting with coordinators Greg Kattner (M118) and William Orrick (M119), the committee selected questions from final examinations. The committee decided that measuring performance on these questions (i) utilizes an existing instrument and thus makes implementation logistically feasible, and (ii) achieves an appropriate balance between accurately measuring student proficiency in the course-specific LOs and falling within the scope of our available resources of time and human effort.

The MathGenEd committee continues its work toward piloting assessment in the remaining MM courses. The MathGenEd committee is slowly increasing the richness of its data collection.

- LOs for variants of M118 (A-, V- and D-) will be the same as for M118.
- LOs for J113 (a variant of M119) will be the same as for M119.
- LOs for M211 are currently in draft, pending e.g. comments by faculty.
- LOs for M213 will be the same as for M211.
- Beginning in Fall 2011, for courses with departmental (common) midterms, data will be collected from both midterm and final examinations, with the recognition that not all course-specific learning objectives are yet addressed in the course syllabus at the time the midterm exam takes place.
III. Internal course-specific learning objectives

Math 118

Learning objectives for Mathematics 118 include but are not limited to the following:

1. Students should become proficient in using combinatorics and probability to model problems in a variety of applied areas. This includes identifying which problems can be solved using such methods, solving the resulting mathematical problems, and drawing qualitative conclusions from the numerical solutions.

2. Students should become proficient in modeling using systems of linear equations in a variety of applied areas. This includes creating variables, translating information about the relationships among these variables into linear equations, incorporating other given data, solving the resulting mathematical problems, and drawing qualitative conclusions from the numerical solutions.

3. Students should become proficient in modeling linear decision-making problems in settings drawn both from business and from everyday experience. This includes creating variables, translating given constraint information into linear inequalities, incorporating given data, solving the resulting linear optimization problem, and deducing optimal decision choices by analysis and by graphical representation of the constraints.

Examples

1. A test for use of a certain illegal drug is 95% accurate, which means that 95% of users will test positive and 95% of nonusers will test negative. It is known that, in the broad population under consideration, 3% are users. Suppose that an individual tests positive. How likely is that he actually is a user? If this is a test taken by college athletes for use of performance-enhancing drugs, would you exclude this individual from competition? If this is a test taken by airline pilots for use of impairment-inducing drugs, would you board the airplane?

2. $20,000 is to be invested in some combination of bank CD’s, bonds, and equities. CD’s are earning 1% simple interest per year, bonds are earning 3%, and on a risk-adjusted basis, equities are expected to yield 5%. How much should be invested in each so that the anticipated yields after one year will all be the same?

3. A health care facility is forming teams of doctors and nurses to examine and inoculate the members of a large population against a certain contagious disease. A “full team” consists of one doctor and three nurses and can treat 180 people per hour; a “half team” consists of one doctor and two nurses and can treat 100 people per hour. There are 200 doctors and 450 nurses available. How should the teams be configured to treat the largest number of people per hour if it is predetermined that there must be be at least 50 full teams and 50 half teams?
Learning objectives for Mathematics 119 include but are not limited to the following:

1. Students should become proficient in modeling problems from a variety of applied areas using linear, exponential, and logarithmic functions. This includes identifying which problems can be solved using such models, creating variables, deducing relationships, solving the resulting mathematical problems, and drawing qualitative conclusions from the numerical solutions.

2. Students should become proficient in calculating, estimating, expressing, and interpreting average, relative, and instantaneous rates of change of one quantity with respect to another, using the language of differential calculus. This includes situations when the relationship between the quantities takes the form of a table, a graph, a textual description, or a symbolic formula.

3. Students should become proficient in modeling optimization problems in a variety of applied areas. This includes creating independent and dependent variables, translating constraint information into an interval of values for the independent variable, solving the resulting optimization problem using techniques from differential calculus, and drawing qualitative conclusions from the numerical solutions.

4. Students should become proficient in calculating, estimating, expressing, and interpreting the accumulated change of one variable, given its rate of change with respect to another variable, using the language of integral calculus. This includes situations when the relationship takes the form of a table, a graph, a textual description, or a symbolic formula.

Examples

1. Owing to an innovative public health program, infant mortality in Senegal, West Africa, is being reduced at a rate of 10% per year. About how long will it take for infant mortality to be reduced by 50%?

2. The table below gives the percent of the US population living in urban areas as a function of the year.

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<thead>
<tr>
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</tbody>
</table>

   (a) Find the average rate of change of the percent of the population living in urban areas between 1890 and 1990. (b) Estimate the derivative of this function at the year 1990 and interpret your answer as an instantaneous rate of change, giving appropriate units.

3. At a price of $8 a ticket, a group can fill every seat in a hall with a capacity of $1500. For every additional dollar charged, the number of people buying tickets is predicted to decrease by $75. According to this prediction, what ticket price maximizes revenue?

4. At what constant continuous rate must money be deposited into an account earning interest at an annual rate of 8.5% compounded continuously in order to save $20,000 in 15 years?
Math 211 (proposed, not yet confirmed by MathGenEd)

Learning objectives for Mathematics 211 include but are not limited to the following:

1. Students will demonstrate fluency in the language of functions. Specifically: they will demonstrate proficiency in (i) recognizing, calculating with, and modeling with polynomial, rational, algebraic, exponential, logarithmic, and trigonometric functions; (ii) recognizing, calculating with, and modeling with inverse functions other than trigonometric; (iii) recognizing, calculating with, and interpreting limits of functions, including limits at infinity; (iv) recognizing, establishing, and applying continuity of functions.

2. Students will demonstrate proficiency in calculating, estimating, expressing, and interpreting average, relative, and instantaneous rates of change of one quantity with respect to another, using the language of differential calculus. This includes situations when the relationship between the quantities takes the form of a table, a graph, a textual description, and a symbolic formula.

3. Students will demonstrate proficiency in modeling optimization problems in a variety of contexts, both applied and abstract. This includes creating independent and dependent variables, translating constraint information into an interval of values for the independent variable, solving the resulting optimization problem using techniques from differential calculus, and drawing qualitative conclusions from the numerical solutions.

4. Students will demonstrate proficiency in calculating, estimating, expressing, and interpreting the accumulated change of one variable, given its rate of change with respect to another variable, using the language of integral calculus. This includes situations when the relationship takes the form of a table, a graph, a textual description, or a symbolic formula.

5. Students will demonstrate proficiency in the expression of mathematical reasoning by stating, applying in appropriate problems, and interpreting the major milestone theorems of one-variable calculus, specifically: the Intermediate Value Theorem, the Mean Value Theorem, and the Fundamental Theorem of Calculus.
Examples

1. The theory of relativity predicts that the mass in kilograms, m, of a body travelling at velocity v meters per second is given by the formula

\[ m = \frac{m_0}{\sqrt{1 - (v/c)^2}} \]

where \( m_0 \) is its mass at rest and \( c \approx 10^8 \) meters per second is the speed of light. What does this theory say as the velocity of an object approaches the speed of light?

2. In 1999, the population of the Richmond-Petersberg, Virginia, metropolitan area was 961,400 and rising at roughly 9200 people per year. The average annual income was $30,593 per capita, and this average was increasing at about $1400 per year. Estimate the rate at which total personal income (the product of population and per capita income) was rising in 1999.

3. A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can she do this so as to minimize the cost of the fence?

4. A model for the metabolism rate, in Calories per hour, of a young man is

\[ R(t) = 85 - 0.18 \cos(\pi t/12) \]

where \( t \) is the time in hours measured from 5 AM. What is the total basal metabolism of this man, defined as

\[ \int_0^{24} R(t) \, dt \]

over a 24-hour period?

5. If \( \frac{dw}{dt} \) is the rate of growth of a child in pounds per year, what does \( \int_5^{10} \frac{dw}{dt} \, dt \) represent? In your explanation, cite an appropriate theorem, and why the hypotheses are satisfied.

Note: The definition of MM courses includes those that are “either required for students in the natural and mathematical sciences, or...”. The LOs #1 and #5 listed above clearly place some emphasis on topics that are perhaps not arguably modeling, but which are arguably prerequisites for further study in mathematics and the sciences.
IV. Mapping internal course-specific learning objectives to GenEd MM learning objectives

It is clear from the text of the given examples that a correct solution will involve the creation of mathematical models, will be expressed using variables and other abstractions, and will require an inference relating conclusions regarding the mathematics to the context of the problem.

V. Spring 2011: instruments and data

Math 118

The following question from the final examination was selected to measure **LO #1:**

*There are 4 black mice and 4 white mice available for an experiment that requires 4 mice. If the 4 mice are chosen simultaneously and at random, what is the probability that 1 is white and the other 3 are black?*

The following question from the final examination was selected to measure **LO #2:**

*At present, City A has a population of 2000 people and is growing at the rate of 100 people per year while City B has a population of 1000 people and is growing at the rate of 300 people per year. In how many years will these two cities have the same population?*

The following question from the final examination was selected to measure **LO #3:**

*Tim’s Sandwich Shop makes large and small sandwiches. One large sandwich uses 10 inches of bread and 8 ounces of meat. One small sandwich uses 6 inches of bread and 5 ounces of meat. Each day Tim’s has available 100 feet of bread and 50 pounds of meat. If the profit on one large sandwich is $1.50 and the profit on one small sandwich is $1.25, then how many sandwiches of each size should Tim’s make to maximize their daily profit?*

*(1 foot = 12 inches) (1 pound = 16 ounces)*

The table below summarizes the corresponding student performance. Listed are percentages of students in each of the listed sections who answered the corresponding question correctly. Data is unavailable for some sections, indicated with “NR”.

<table>
<thead>
<tr>
<th>LO</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
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<td>90.6</td>
<td>94.2</td>
</tr>
</tbody>
</table>
Math 119

The following question from the final examination was selected to measure **LOs #1:**

An oil tanker has run aground and is leaking oil. The function $V(t)$ represents the quantity of oil remaining in the tanker, measured in thousands of barrels, as a function of time, $t$, measured in days since the accident. After 12 days, it was observed that $V(12) = 365$ and $V'(12) = -9$. Using local linear approximation, estimate the amount of oil remaining in the tanker 14 days after the accident.

The following question from the final examination was selected to measure **LOs #2:**

A theater owner finds that the ticket price $p$ that would result in a demand for $q$ theater tickets is given by $p = 60 - q/40$. As the capacity of the theater is 1100, the quantity $q$ is restricted to the interval $[0, 1100]$. What is the ticket price that would maximize revenue?

The following question from the final examination was selected to measure **LOs #3:**

A ball is thrown vertically upward by a student standing on the balcony in the IU Auditorium. Take the displacement of the ball as it is released to be 0 meters, with positive displacement indicating that the ball is above balcony level, and negative displacement indicating that the ball is below balcony level. The velocity of the ball, in meters per second, is described by the formula $v(t) = 14 - 10t$, where $t$ represents time, in seconds, since the ball was thrown. Find the displacement of the ball three seconds after it was thrown.

The following question from the final examination was selected to measure **LOs #4:**

Cell phone service was first established in a certain community at the start of 1993. The number of cell phone users in that community as a function of time in years since the start of 1993 is well described by the logistic function $P(t) = 40,000/(1 + 41 \exp(-0.35t))$. In what year did the rate of growth in the number of users peak? (Answer should be a whole number, and should be a year, e.g. 2011.)

The table below summarizes the corresponding student performance. Listed are percentages of students in each of the listed sections who answered the corresponding question correctly.

<table>
<thead>
<tr>
<th>LO</th>
<th>Section (internal reference code)</th>
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</table>
VI. Reflecting on the process and the data

- The Math GenEd committee maintains richer data regarding GenEd assessments when available.
- Communication between it and the coordinators, and the coordinators and instructors, must improve.
- The Math GenEd committee has not yet formally met to draw inferences from the Spring 2011 data.
- The committee has concerns about balancing the need to maintain stability of the assessment instruments from year to year with the security of examinations.
- The committee has concerns about the extra effort needed to process the additional data and to compile the requested reports.